The topics are divided into six general areas:

1. Group Theory
   - Nilpotent groups
   - Free groups
   - Linear groups
2. Category theory
   - Categories, subcategories
   - Functors, equivalence of categories
   - Adjoint functors
   - Universal properties, representability
3. Galois theory
   - Algebraic closure
   - Algebraic, normal and separable extensions
   - Galois correspondence (finite extensions)
   - Solvability of equations
   - Cubic and quartic equations; cyclotomic fields
4. Field theory
   - Algebraic and transcendental extensions
   - Transcendence basis of an extension
5. Commutative ring theory
   - Localization; support of a module
   - Spectrum of a commutative ring
   - Noetherian and Artinian rings
   - Hilbert Nullstellensatz
   - Hilbert Basis Theorem
   - Integral extensions; integral closure
   - Associated primes of a module
   - Discrete valuation rings; Dedekind domains
   - Projective, injective and flat modules; invertible ideals
6. Noncommutative ring theory
   - Tensor products
   - Tensor, symmetric and exterior algebras
   - Primitive rings; density theorem
   - Semisimple rings
   - Wedderburn's theorem on finite division rings

Bibliography:

In recent years, one of the following has served as the core text for the course.

2. Thomas W. Hungerford, Algebra, Springer Graduate Texts in Mathematics 73
3. Serge Lang, Algebra, Springer Graduate Texts in Mathematics 211

Supplementary material can be drawn from the following books.

1. Irving Kaplansky, Fields and Rings,
   (supplementary material for Galois theory, particularly cubic and quartic equations and cyclotomic fields; supplementary material for noncommutative rings, particularly Noetherian and Artinian rings, the Hilbert Nullstellensatz and Hilbert Basis Theorem)
2. Hideyuki Matsumura, Commutative Ring Theory, Cambridge University Press (supplementary material in the area of commutative ring theory, particularly the first two and last three subtopics)

Additional Resources:

- Richard Foote's page of errata for Dummit and Foote
- George Bergman's Companion to Lang's Algebra with errata
- GMA page on past PhD Algebra Exams

(posted April 11, 2014)