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MAP 6505 Mathematical Methods in Physics and Engineering I: Syllabus

Prerequisites

(i) Multi-variable Calculus, Linear algebra, and Differential equations (**necessary**),
(ii) Introduction to Partial Differential Equations (**recommended**),
(iii) Advanced Calculus or Mathematical Analysis or their equivalents (**can be helpful, but not necessary**).
No permission for graduate students is required to enroll. Graduate students decide for themselves how well they know prerequisites. Undergraduate students willing to take this course have to get a permission from the department (please email me on this matter indicating UF mathematics courses you have taken).

Recommended Additional Texts

L. Schwartz, Mathematics for Physical Sciences,
V.S. Vladimirov, Equations of Mathematical Physics,

Lecture Notes

S.V. Shabanov, Distributions and Operators for Theoretical Physicists (to be posted in the course page; they are self-sufficient).

Goals of the course

The aim of the course to teach basics mathematical skills which are necessary for analytical calculations and analysis of physics theories and mathematical models in engineering, especially calculations involving distributions, Green's functions, and distributional solutions to equations of mathematical physics. The course is especially useful for students specializing in theoretical and mathematical physics.

Course Content

Topic 1: **Review of Integration Theory.** Functions in Euclidean spaces. Review of the Riemann integration theory in Euclidean spaces. Absolutely and conditionally convergent improper integrals. Line and surface integrals. Divergence theorem and integration by parts in Euclidean spaces. The Lebesgue theory of integration. The space of Lebesgue square integrable functions. Lebesgue integrals depending on parameters. Integration and differentiation with respect to parameters. Potential type integrals and Gaussian integrals. Improper Lebesgue integrals depending on

parameters.

Topic 2: **The theory of distributions** (the main part of the course). Test functions and linear functionals on them. Distributions. Regular distributions vs ordinary functions. Dirac delta-functions and other singular distributions. Calculus with distributions. Relations between classical and distributional derivatives. Series with distributional terms. Poisson summation formula. Direct product and convolution of distributions. Tempered distributions. Fourier transform of tempered distributions.

Topic 3: **Distributional solutions to linear differential equations and Green's functions**. Basics equations of mathematical physics and their distributional solutions. Fundamental solutions for linear differential operators (Laplace, Helmholtz, wave, heat, and Schroedinger operators in various dimensions). Finding a fundamental solution by the Fourier method. Causal Green's functions and the Cauchy (initial value) problem for basic equations of mathematical physics (heat, wave, and Schroedinger equations). Wave potentials and wave propagation in various dimensions. Green's function for a Helmholtz operator satisfying Sommerfeld radiation conditions and applications to wave scattering (time permitted).

Lectures

There will be three lectures per week. A brief description of each lecture will be posted in **the course page** along with recommended texts useful for further reading on the topic discussed. There are video lectures (recorded in the first year of the pandemic). They will be available to watch through the links in the course page. However, they can differ significantly from the lectures in class because the content has been adjusted in the past years, Posted lecture notes are close to the actual lectures. Students are expected to read some parts of the notes not covered in class. Some facts from Mathematical Analysis and Functional Analysis will be given without proofs. The references where the proofs can be found are given in the Lecture Notes.

Graded assignments

Homework: Practice problems can be found in Lecture Notes. They are not mandatory but doing them would be of great help to do well on graded assignments. Practice problems will be assigned during the lectures and some of them will be included into graded assignment.

Graded assignments: There will be 3-4 graded homework assignments based on Exercise Sections of the Lecture Notes. Submission is via Canvas. All assignments are due in a week or so after they are open in Canvas. They are free-response. Pick any problem from the assignment, mark its number, write your solution with ALL technical details clearly indicating how the final answer is obtained. Box the final answer. For example, if you use a particular theorem in your solution, make sure that the hypotheses of this theorem are shown to hold (otherwise no credit). Do the same for other problems problems. Enumerate all pages as 1/n, 2/n, …, n/n where n is the total number of pages in your submission. In the bottom of the last page write and sign the academic honesty pledge (the text will be provided with the exam). Scan your work into a single PDF file and name the file as LastNameFirstNameHW# (where # is the assignment number, 1, 2, etc.) and submit via Canvas.

Grading

Each assignment contains some number of problems and each problem is worth a point if solved correctly. If N is the total number of assigned problems and M is the total number of points earned, then your current score is G=100(M/N) rounded to the nearest integer. The grade thresholds are:

A: G>90; A-: G>85; B+: G>80; B: G>75; B-: G>70; C+: G>65; C: G>60; C-: G>55; D+: G>50; D: G>45; D-: G>40; E: G<40

Extra credit: Occasionally an extra credit problem will be added to an assignment. It does not increase N, but it would increase M if solved correctly.

Office Hours

You can ask questions about course topics and homework assignments after every lecture (6-7 pm) in class. Otherwise make an appointment by email for either in-person or Zoom meeting.

Policies

Attendance: No credit for attendance.

Special accommodation: Students requesting special accommodation for exams must first register with the Dean of Student Office. The Dean of Student Office will provide documentation to the student who must then provide this documentation to me when requesting accommodation.

Student honor code: When turning in any graded assignment, please write “I did the assignment myself and received no help from anybody” and sign it. Assignments turned in after the due date and/or without a signed academic honesty pledge will not be accepted.