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College *of* Liberal Arts *and* Sciences

MAP 6506: Mathematical Methods in Physics II; Syllabus

Class 2025 (Spring)

Prerequisites

UF Calculus 3, Linear Algebra, and Introduction to Partial Differential Equations are necessary. UF Advanced Calculus or Mathematical Analysis or their equivalents and Part I of this course are very helpful, but not mandatory. However, basic concepts of the theory of distributions and the Lebesgue integration theory have to be reviewed in order to understand the beginning of Part II (Green's functions for differential operators). The main part of Part II, the theory of operators in Banach and Hilbert spaces, is not based on Part I. Students who did not take Part 1 of the course are advised to review some basic topics on distributions using Chapter II (Sections 5-8) of the textbook by Vladimirov or read my lecture notes (the links can be found in the course page). There is a section for advanced undergrads. But admission only by the consent of the instructor. Undergraduate students should contact the instructor on this matter.

Recommended Texts

V.S. Vladimirov, Equations of Mathematical Physics (for reviewing topics of part I)
F. Riesz and B. Sz.-Nagy, Functional Analysis
I. Stakgold, Green's functions and boundary value problems (main textbook),
M. Reed and B. Simon, Methods of Modern Mathematical Physics, Vol 1-4 (for advanced reading).
S.V. Shabanov, Lecture Notes on Mathematical Methods in Physics (will be posted in the course page)

Course Content

Part 1: Cauchy (initial) value problem for basic evolution equations in physics (wave, heat, Schroedinger, and radiative transfer). Wave propagation and scattering, the Lippmann-Schwinger (integral equation) formalism. Applications include sound waves, electromagnetic waves, and quantum mechanical scattering. Scattering on small particles. Perturbation theory formalism.
Part 2: The theory of integral equations in Banach spaces. Green's functions for differential operators and related boundary value problems for the Laplace and

Poisson equations. Integral equations used in image analysis (in medicine and plasma physics) based on Abel's and Radon transforms. Volterra and Fredholm integral equations. Von Neuman series. Perturbation theory.
Part 3: Operators in Banach and Hilbert spaces. Bounded and unbounded operators. The inverse and adjoint operators. The resolvent of an operator. Fredholm alternative for a general linear equation in a Hilbert space. Spectrum of an operator. The theory of self-adjoint operators in a Hilbert space. The spectral theorem.
Applications to quantum mechanics and Fourier methods for integral and partial differential equations.

Class meetings, attendance, and office hours

The class will meet on MWF in person. Attendance is not mandatory but strongly recommended. Each Monday and Friday, there will be a discussion session right after the class meeting, provided there are questions to be discussed.

Exams

There will be 3-4 graded assignments and the final exam. The graded assignments will be conducted via Canvas every other week or so. Each assignment covers the material given prior the assignment. Each assignment is open for a specified period of time (typically several days) during which it must be completed and submitted via Canvas. The submission is free-response. Indicate the problem number, write your solution (do not omit technical details), box the answer, do the same for all problems, enumerate all pages as 1/n, 2/n, …, n/n, where n is the total number of pages, write and sign the academic honesty pledge at the bottom of the last page, write your name and your UFID number, scan all the pages in the above order into a single PDF file, and submit the file via Canvas. Make sure that you have a software or app to make such a PDF file. Other formats are not acceptable. Late submissions will not be accepted. Topics for the final exam will be announced toward the end of the semester. It will be conducted in the same fashion as regular graded assignments. You may use anything to prepare your submissions.

Special accommodation: Students requesting special accommodation for exams must first register with the Dean of Student Office. The Dean of Student Office will provide documentation to the student who must then provide this documentation to me when requesting accommodation.

Student honor code: Each submitted assignment must contain the signed academic honesty pledge: "Herewith I acknowledge that I did all the above problems myself and did not receive any help from any person". Submissions without the signed honesty pledge will not be accepted. You are NOT allowed to discuss any assignment during the time period the assignment is open on Canvas. A breach of this policy is considered as cheating. If caught cheating, the course grade is an F, no exception.

Homework

Lecture notes contain practice problems. Solving these problems is essential for understanding the course and attaining a good grade. Some of the homework problems can be discussed during office hours

Grading

Each assignment contains some number of problems. Each problem is worth one point if solved correctly. If M is the total number of earned points and N is the total number of regular problems given, then your current grade is the average:

G = (M/N) 100%

The grade thresholds are:

A: G>90; A-: G>85; B+: G>80; B: G>75; B-: G>70; C+: G>65; C: G>60; C-: G>55; D+: G>50; D: G>45; D-: G>40; F: G<40

Extra credit: There will be extra credit problems given in some of the assignments. They are not counted in the number N, but can increase your number M if solved correctly. The perfect score can therefore exceed 100% when the extra credit questions are correctly answered.