Krishnaswami Alladi
Department of Mathematics

College of Liberal Arts and Sciences

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MAS 7215 and MAT 6932 – Number Theory – Irrationality, Diophantine Approximations and Transcendence

Irrationality, Diophantine Approximations and Transcendence

NOTE: The course is designated as MAS7215 (Section 6571) and MAT 6932 for convenience in enrollment.

MWF 7th period (1:55-2:45pm) – LIT 221 – FALL 2024

INSTRUCTOR:

Krishnaswami Alladi
304 Little Hall
(352) 294-2290
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OFFICE HOURS:

M and W 9th period (4:05 – 4:55 pm) in LIT 304 and by appointment.

PREREQUISITES:

Undergraduate course in number theory and a course in complex variable theory

SPECIFIC TOPIC FOR COURSE:

Irrationality, Diophantine Approximations and Transcendence

COURSE DESCRIPTION:

The study of irrational numbers dates back to Greek antiquity. Yet the subject remains an active area of research today. Although "almost all" real numbers are irrational, it is very difficult to establish the irrationality of a given number. We will systematically study a variety of techniques which will help in confirming the irrationality of different classes of numbers starting with Dirichlet's fundamental criterion for irrationality. Following this, we will discuss irrationality criteria utilizing series and product representations of reals due to Engel, Cantor and Sylvester, regular and general continued fraction expansions, and Farey fractions. We will present various proofs of the irrationality of important numbers such as e and pi, and the irrationality of functions like the trigonometric, hyperbolic and the Bessel functions at rational arguments. Next we will take up the study of the closeness of approximation of irrationals by rationals and introduce the concept of irrationality measures. This is the subject of Diophantine approximations.

It will be shown that the truncations of the regular continued fraction expansion of real numbers generate the sequence of "best approximations", but it is very difficult to determine the continued fraction expansion of a given irrational.

We will develop methods to obtain efficient irrationality measures. This will involve the use of Legendre polynomials and Pade approximations. An important outcome of this approach is the irrationality of the Riemann zeta function at the odd integer 3 – a fact long conjectured but only established in 1978. The study of irrationality measures also explains why Pell's equation such as $x^2 - 2y^2 = 1$ has infinitely many solutions whereas the Thue equation: $x^3 - 2y^3 = 1$ has only finitely many integer solutions. Thus the subject of Diophantine approximations is closely associated with the theory of Diophantine equations. As part of our study of Diophantine approximations, we will include a discussion of uniform distribution and of normal numbers – numbers whose decimal digits are statistically uniformly distributed.

Finally, we will launch the study of transcendental numbers by showing first that e is transcendental, and more generally that the exponential function takes transcendental values at non-zero algebraic arguments. Since exp $(i\pi)=-1$ this result of Lindemann at the end of the 19-th century implies that π is transcendental and thereby shows the impossibility of "squaring the circle" – one of three problems of Greek antiquity. We also hope to discuss measures of transcendence and the transcendence of some fascinating functions (defined by lacunary series) at algebraic arguments.

A host of great problems remain unsolved:

Is Euler's constant γ irrational?

Are the values of the Riemann zeta function at the odd integers >3 irrational?

What is the precise irrationality measure for π?

We know that "almost all" numbers are normal to all bases, but we do not know a single example of a number whose digit expansions to all integer bases (not just base 10) are normal.

The course will be completely self contained and can be followed by any of our graduate students and those in allied disciplines as well (such as computer science and statistics). I will use my own detailed notes for the course but will give a number of books as references.

The list of topics given above is vast, and I will cover as many of them as time permits and depending on the interest of the audience. My goal is not to race through the topics, but do a thorough discussion at a pace that is comfortable. If there is interest, this course might be extended into Spring 2025 as MAS 7216 and MAT6932

The course will be self contained and should appeal to any graduate student or advanced undergraduate.

TEXT:

There will be no assigned text. I will use my own notes. A number of texts will be given as references.

GRADING:

Grades will be based on homework that will be assigned periodically.

ACCOMODATION FOR STUDENTS WITH DISABILITIES:

Students requesting classroom accommodation must first register with the Dean of Students

Office. The Dean of Students Office will provide documentation to the student who must then provide this documentation to the Instructor when requesting accommodation.

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