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Department of Mathematics

MAT 6932/3F93 depx/16134 Special Topics in Applied Mathematics (Spring 2021)

Introduction to Deep Learning and Its Mathematical Foundation (II): “Deep Learning Methods, Theories and Applications”

Instructor:

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Objective and Description of the Course:

> Deep learning (DL) is a novel methodology currently receiving much attention and has been successfully applied to a large variety of fields, such as data classification, image recognition, speech recognition, natural language understanding, precision medicine, and computational biology. The overall aim of this course is to prepare our students this modern computational skill for his/her future research. Based on the basic concept of statistical learning theory and knowledge on deep neural network (DNN) architecture and training that we studied in Fall 2020, this course will focus on the study of the architectures and training of the DNNs inspired by variational methods, optimization algorithms, ordinary differential equations and optimal control techniques, and their applications to image reconstruction, transformation, classification, segmentation and meta-learning. We will go through numbers of case study to compare the advantage and dis-advantage of different designs in those applications.
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> To better understand the theoretical foundation of the DNNs inspired by variational methods, optimization algorithms and optimal control techniques this, this course will introduce some necessary knowledge on convex and nonconvex analysis and several popular optimization algorithms for solving certain classes of convex/nonconvex and smooth/nonsmooth optimization problems and optimal control problems arising from deep learning approaches.

Course Material: The topic of this course is one of the rapid developing fields. There is no textbook available. References (recent papers) related to the topics of the course are provided below.

References:

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Meeting Time and Rooms*:*

MWF 4 online (Synchronous)

Office Hours: MWF 5 or by appointment

Arrangement of the Course:*:*

Unit 1: Basic concepts on convex and nonconvex analysis: (Tentatively week 1-4)

2.1. Convex functions: Definition and basic properties of convex functions, convex conjugate function, properties and calculation of conjugate function;(weeks 1-1.5)

2.2. Functions with Lipschitz continuous gradient: Definition and properties; (weeks 1.5-2)

2.3. Nonsmooth convex and non-convex functions: Sub-gradient and sub-differential for nonsmooth convex function – definition and basic properties, sub-gradient calculus, duality and optimality conditions; Generalized directional derivatives and subgradients of nonconvex nonsmooth functions; (week 3-4)

Unit 2: Variational models and first order optimization algorithms inspired deep neural networks and their applications: (Tentatively week 5-9)

2.1. Gradient decent method, proximal gradient method, accelerated gradient decent methods for unconstrained convex and set constrained convex and nonconvex optimizations: Optimality conditions, local minimum and global minimum, schemes, shrinkage operator and convergence analysis; (weeks 5-6)

2.2. Proximal gradient and gradient decent algorithms inspired convolutional networks for solving inverse problems: Architecture and training of the CNNs for solving inverse problems, case study: Prox-net, ISTA-net, OPINE-net, Neumann-net and Variational-net; (weeks 6-7)

2.3. Alternating direction method of multipliers (ADMM) for equality constrained convex optimization and ADMM inspired CNNs: Architecture and training of the CNNs for solving inverse problems, case study: ADMM-net; (week 8)

2.4. Primal-dual algorithm and Primal-dual algorithm inspired CNNs: Architecture and training of the CNNs for solving inverse problems, case study: PD-nets; (week 9)

Unit 3: Variational methods and variational CNN for image segmentation: (Tentatively week 10-11)

3.1. Mumford-Shah model, Chan-Vese model and Potts model for image segmentation/partition; (week 10)

3.2 Variational CNN for image segmentation and case study: Total variation and soft threshold dynamics regularized variational networks for image segmentation, case study; (week 11)

Unit 4: Deep learning from optimal control perspective: (Tentatively week 12-16)

4.1. Brief introduction mathematical control theory: Controlled dynamics, Calculus of variation, Hamiltonian dynamics, Pontrygin Maximum Principle (PMP), free time fixed endpoint problem, applications; (weeks 12-13)

4.2.Deep neural network and numerical ODE: Connection of discrete dynamical systems with DNN, Neural ODEs for supervised learning and case study: Neural Ordinary Differential Equations; (week 14)

4.3. Deep Learning as optimal control problems: Representability and controllability Problem, Maximum Principle based models and numerical methods, applications for image classification, restoration and reconstruction; (week 15-16)

Additional Information:

Grading:
Students will be required to a presentation related to the course content or problems of particular interest to the individual student. Grades will be assigned on the basis of the presentation. More information on UF grading policy may be found at:
UF Graduate Catalog
Grades and Grading Policies

Honor Code: “UF students are bound by The Honor Pledge which states, “We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honor and integrity by abiding by the Honor Code. On all work submitted for credit by students at the University of Florida, the following pledge is either required or

implied: “On my honor, I have neither given nor received unauthorized aid in doing this assignment.” The Honor Code specifies a number of behaviors that are in violation of this code and the possible sanctions. Furthermore, you are obligated to report any condition that facilitates academic misconduct to appropriate personnel. If you have any questions or concerns, please consult with the instructor or TAs in this class.”

Class Attendance: “Requirements for class attendance and make-up exams, assignments, and other work in this course are consistent with university policies that can be found at: https://catalog.ufl.edu/ugrad/current/regulations/info/attendance.aspx.”

Accommodations for Students with Disabilities: “Students with disabilities requesting accommodations should first register with the Disability Resource Center (352-392-8565, https://www.dso.ufl.edu/drc/) by providing appropriate documentation. Once registered, students will receive an accommodation le

Online Evaluations: “Students are expected to provide feedback on the quality of instruction in this course by completing online evaluations at https://evaluations.ufl.edu. Evaluations are typically open during the last two or three weeks

Contact information for the Counseling and Wellness Center: https://counseling.ufl.edu/, 392-1575; and the University Police Department: 392-1111 or 9-1-1 for emergencies.

Diversity:
I and the department of Mathematics are committed to diversity and inclusion of all students in this course. I acknowledge, respect, and value the diverse nature, background and perspective of students and believe that it furthers academic achievements. It is our intent to present materials and activities that are respectful of diversity: race, color, creed, gender, gender identity, sexual orientation, age, religious status, national origin, ethnicity, disability, socioeconomic status, and any other distinguishing qualities.

Class Recording:
Our class sessions may be audio-visually recorded for students in the class to refer back and for enrolled students who are unable to attend live. Students who participate with their camera engaged or utilize a profile image are agreeing to have their video or image recorded. If you are unwilling to consent to have your profile or video image recorded, be sure to keep your camera off and do not use a profile image. Likewise, students who un-mute during class and participate orally are agreeing to have their voice recorded. If you are not willing to consent to have your voice recorded during class, you will need to keep your mute button activated and communicate exclusively using the “chat” feature, which allows students to type questions and comments live. The chat will not be recorded or shared. As in all courses, unauthorized recording and unauthorized sharing of recorded materials by students or any other party is prohibited.

Software Use
All faculty, staff, and students of the University are required and expected to obey the laws and legal agreements governing software use. Failure to do so can lead to monetary damages and/or criminal penalties for the individual violator. Because such violations are also against University policies and rules, disciplinary action will be taken as appropriate. We, the members of the University of Florida community, pledge to uphold ourselves and our peers to the highest standards of honesty and integrity.

Student Privacy

There are federal laws protecting your privacy with regards to grades earned in courses and on individual assignments. For more information, please see the Notification to Students of FERPA Rights.

Campus Resources:

Health and Wellness

U Matter, We Care:
If you or a friend is in distress, please contact umatter@ufl.edu or 352 392-1575 so that a team member can reach out to the student.

Counseling and Wellness Center: counseling.ufl.edu/cwc, and 392-1575; and the University Police Department: 392-1111 or 9-1-1 for emergencies.

Sexual Assault Recovery Services (SARS)
Student Health Care Center, 392-1161.

University Police Department at 392-1111 (or 9-1-1 for emergencies), or police.ufl.edu.

Academic Resources

E-learning technical support, 352-392-4357 (select option 2) or e-mail to Learning-support@ufl.edu.

Career Resource Center, Reitz Union, 392-1601. Career assistance and counseling.

Library Support, Various ways to receive assistance with respect to using the libraries or finding resources.

Teaching Center, Broward Hall, 392-2010 or 392-6420. General study skills and tutoring.

Writing Studio, 302 Tigert Hall, 846-1138. Help brainstorming, formatting, and writing papers.

Student Complaints Campus

On-Line Students Complaints

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