Sets and Logic

Nostalgic?   See 2022t, 2021t, 2021g, 2020t, 2020g, 2019t, 2018t, 2017g, 2015g, 2014g, 2013t, 2012g, 2011t, 2009t, 2008g *and* current SeLo.

Our Teaching Page has important information for my students. (It has the Notes, Exams and Links from all of my previous courses.)

The *Teaching Page* has **my schedule**, LOR guidelines, and Usually Useful Pamphlets. One of them is the *Checklist* (pdf) which gives pointers on competant mathematical writing. Further information is at our class-archive URL (I email this private URL directly to students).

> In all of my courses, **attendance is absolutely required** (excepting illness and religious holidays). In the unfortunate event that you miss a class, *you are responsible* to get all Notes / Announcements / TheWholeNineYards from a classmate, or several. All my classes have a **substantial** class-participation grade.

Assignment for first week of SeLo: (See below, for the materials referred to.)

- To help you self-evaluate, take 90 minutes to solve as many problem as you can, on this **test of high-school mathematics, with a touch of calculus**.

- *Memorize!* the Math-Greek alphabet (pdf).

- *Work through* BoP, sections 1.1 through 1.9. Write-up (but do not hand-in) at least 3 HW problems from pages 1-14.

- Read w:Set-builder notation (up through "Equivalent predicates..."), becoming comfortable with the notation.

- In *PList:*, read pages 1-6, memorize abbreviations in *Appendix: Notation*.

- Exams from previous SeLo incarnations:

  This will help you decide if my teaching-style is the right style for you.

- In *SaP* ("Structure and Proof") work through pages 11-17.

  Important: For us, the (double-bar N) symbol $\mathbb{N}=\{0,1,2,...\}$; i.e *zero is a natural number*, a *natnum*. This is also the convention in *SaP* but, unfortunately, not the convention in *BoP*.
  So when you read $\mathbb{N}$ in *BoP*, replace it with $\mathbb{Z}_+=\{1,2,3,4,...\}$; the set of *positive integers*; the *posints*.

Our two, free, online texts (you can freely download the PDFs to your computer) are these:
Main textbook: The Book of Proof (**BoP**), by Richard Hammack.
       Secondarily, we will use
Transition to Higher Mathematics: Structure and Proof (**SaP**), by Bob A. Dumas and John E. McCarthy.

SeLo Resources

- *Memorize!* the [Math-Greek alphabet (pdf)](#).

- [The PList:](#) (Problem List for SeLo has hyperlinks in the Table-of-Contents and the Index.) `UPDATED` [Date].

- [Does Zero = One? (pdf)](#). Here are some ~~proofs~~ *poofs* about which you can post to our Archive.

- *Look Ma!* All **0** SeLo quizzes so far (pdf) `UPDATED` [Date]

- Optional: [Practice: Binomials, complex arithmetic](#).

- Current: Please work-through [w:Euclidean algorithm](#) (up through "Extended Euclidean..." but skip the proofs) *and* work-through [w:Modular arithmetic](#) (through "Applications").
  The **Euclidean algorithm** can be conveniently applied in table-form; I call this form "Lightning Bolt " because the update-rule looks like a lightning-bolt (used thrice). *Please read* the [Lightning-bolt algorithm (pdf)](#), learning the algorithm, but skipping the proofs.
  Suggestion: Print out on paper (yes, *actual paper*), the [practice sheet for LBolt (pdf)](#) and fill-in the tables.

- Current: The first page of 💡 [Algorithms in Number Theory (pdf)](#), uses LBolt iteratively to compute the GCD of a list of integers, together with its list of Bézout multipliers. Page 2 uses LBolt to solve linear congruences: "Find all x where $33x$ is mod-114 congruent to 18."

- Optional: Examples of [fusing congruences (txt)](#) using LBolt.
  Everybody loves the *Euler-Fermat thm*. Available is [Using EFT to solve $102^{70} + 1 =_{113}= b^{37}$ (txt)](#), from Prof. William Stein's book.

- Current: A std proof of the [Inclusion-Exclusion principle (pdf)](#), together with *Candy-Store*, *Derangement* and *Stirling-number* examples.

- An [Introduction to Isomorphism, via Gambling](#). It asks: *Why 2, when 76 seems correct?*

- What is [Hall's Marriage (Matching) Lemma](#)?

- Future: Our [Primer on cardinality](#).
  In addition to proof by *raster scan*, we can prove that NxN is equinumerous with N via [w:Boustrophedon](#), which can even be [pronounced](#)!

- Optional: Our [Primer on Polynomials](#) has further information on Algebraic Numbers, for the *Curious Ambitious Student*.

- Future, optional: [Number Theory grab-bag (pdf)](#). (I wrote this for a NT class, so we'll need to skip parts.) Optional: A proof of the [Chinese Remainder Theorem (pdf)](#) [CRT], as a ring-isomorphism thm. Proves that Euler phi is a multiplicative fnc. An example of using CRT to count roots of a polynomial

- [2022g SeLo syllabus](#).

Web resources

- [Ring Basics (pdf)](#).

- Fun, challenging problems: [IMO](#) and [USAMO](#) and [HMMT](#) and [Putnam](#).

- The famous [On-line Encyclopedia of Integer Sequences](#), and some [w:OEIS history](#), and a [video with a challenge at the end.](#)

- Truth-table displayer:

- A free site, [Merge PDF](), for merging multiple PDFs into one. I've used this and it worked fine. There are other free ones on the web as well. Use whatever works for you.
  In case you want it, this free [as of 21Dec2020]. site [converts PDFs to PNGs]().

- ## Tips on writing proofs
  - ["How To Write Proofs" (html)](), by Prof. Larry W. Cusick, [INTERMEDIATE]. Examples mostly from Elem. Number Theory; some from Calculus.

  - [Prof. Christopher Heil's page (pdf)]() [4 pages, INTRO]. A well written survey of the structure of proofs. Has one example of induction (recursion).

- Optional: A cute proof that **e is irrational**.

Quantifiers ∀ and ∃ ("for all" and "there exists") are like nitroglycerin, in that one little mis-step leads to the whole thing blowing up in your face.
*   There is no partial credit when it comes to Explosives and Quantifiers.*

-JLF King

Ending in style

- *Prof. King's **Mastery of Zoom** [except for the cigarette].*   [Source unknown]

- An [End-of-Semester *Math Derivation*]().

The various Math czars who help out.

Computer&Projector	CP-Probs	Time	Memory/Telepathy	Whiteboard
Brandon	*Alexa & Diego*	*everyone*	*Anneka*	*everyone*

- *Look Ma!* All 11 SeLo quizzes (pdf) **UPDATED** [Monday, 05Dec2022]

- The delightful Home-A.
  Typed and well-stapled, your team's write-up was due [with all team-members present] at beginning-of-class on [~~Wedn., 28Sep~~ UF closed by hurricane Ian]
    Monday, 03Oct.
  It's first page is the printed problem-sheet, with the blanks filled-in (handwritten is fine).

- The heart-warming Class-A was moved to Monday, 03Oct due to hurricane Ian.
  For a deeper understanding of iterated LBolt, a 3D integer-lattice is convenient.
  Related to the essay question is whether *Tilable* Union *Untilable* can be tilable.

- Our *dodecahedral* Home-B was followed by the nicely well-ordered Class-B.

- Our bijective Class-C.

- [2022t Selo syllabus](#).

End-of-semester SeLo (Individual Optional Project) IOP-D (pdf)

...was due, slid $_u{}^n{}_{d}{}_{e}{}_{r}$ my office door (Little Hall 402, Northeast corner) , no later than

2PM, Friday, 09Dec2022

Autumn 2021, SeLo:

- The spiffy The tilable **Class-A (pdf)** was enjoyed by all.
  Our **Home-A (PDF)** was due date Wednesday, 20Oct2021.

- **Home-B (PDF)**, done individually, was due Monday, 29Nov2021

- An intro to the [Chromatic polynomial of a graph (pdf)](#); [a graphical example of deletion-contraction](#).

- The challenging Individual Project C was due the Thursday after the last day of class.

- [2021t Selo syllabus](#).

The **Spring 2021, SeLo**, page:

- Folks appreciated that our first **Enjoyable quiz A** had no essay question. [Future quizzes will have proofs on them, and students will need to upload a (typically, small) essay as a PDF.]

- Scholars showered the keyboard with roses, admiring the **Pleasant quiz B** with its elegant PHP proof.

- SeLoCitizens were entertained by **Goofy quiz C** starting off with a binomial-identity, and finishing with a nice Induction proof. Cognoscenti cheerfully garnered CP-points by posting *solutions* (not just answers) to our SeLo Archive.

- Cognoscenti marveled at the straightforward induction proof on **Daffy quiz D**.

- [Elegant E](#) was everyone's favorite, as it answered that burning light bulb question...

- Computer-generated Lmino tiling appear in [Pictures of Lmino Tilings (txt)](#). The pictures start about a third of the way down the file. [The top of the file just comprises notes to me, on how to use the code.]

- The popular **Quiz Fantabulous!** was a nice productive way end our pandemic Zoomester.

- [SeLo syllabus](#)

Autumn 2020, SeLo, page, for two **online** (due to COVID) sections:

- Beautifully designed [but unexpectedly eventful due to my unfamiliarity with Canvas], our **Exam-A (PDF)**.

- The engaging **Exam-B (PDF)** went smoothly.

- The Individual End-of-Semester Project was administered via Canvas.

- The **2020t Selo syllabus**.

The **Spring 2020, SeLo**, page:

- Our Class-A and Home-A were entertaining.

- The interesting IP-B (Individual-Project B), folks told me, was challenging.

- All B-teams have emailed me their **Home-B**. Thank you.

- *Look Pa!* All 5 SeLo quizzes (pdf).

- Our **SeLo syllabus (txt)** has clickable links.

End-of-semester SeLo Individual Project D (IP-D)
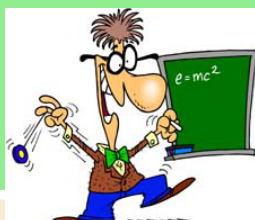
[Problems (D2(C)) and (D3) are challenging. Don't delay, said the Wise Person...]
...was due, emailed to me, squashATuf1DOTedu, as a PDF! no later than 5PM, Tuesday, 28Apr2020.

*T*he SeLo Project must be **carefully typed**, but diagrams may be hand-drawn and scanned into the PDF.

The various SeLo czars who helped:

TIME	COMPUTER	MEMORY/telepathy	SPUR-OTM-PROBS	PHONE
Jonathan S.	Teegan B.	Yasmeen G.	Blake W.	Chris P.

Autumn 2019, SeLo:

- In one convenient location: All 9 SeLo quizzes (pdf),

- The Lminoish **Class-U**
  Our **Home-U** had solns. Related to the exam are Pictures of **Lmino Tilings (txt)**, to illustrate a geometric induction-argument.

- Our **Home-V**.
  SSAY *(V3)* HAD YOU LABEL THE VERTICES A **DODECAHEDRON**. YOU MAY HAVE FOUND IT CONVENIENT TO PRINT OUT THE **EDGE-GRAPH OF A DODECAHEDRON**

Efor your vertex-label diagrams.

Available is a [template, for folding-together a 3D-dodecahedron](#).

- The *bijective* **Class-W**.

- The intriguing **IOP-X** had a Pigeon-hole Principle question, and a cardinality question.

- Our [SeLo syllabus/calendar](#).

The various Math czars who help out:

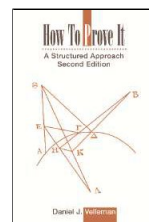| Time	Computer	Memory/telepathy	Blackboard	Spur-OTM-Probs
Chase	Atharva	Sienna	Nathan	Bhaskar

Autumn 2018, SeLo:

Our textbook is *How to prove it, (**Second** Edition)*.

Author: Daniel J. Velleman   **ISBN-13:** 978-0521675994

Year: 2006   **Publisher:** Cambridge University Press

Here are links to [this book at The Publisher's site](#) and at [Amazon.com](#).

***S***eLoidal czars who aided the course:

Time	Computer	Spur-OTM-Probs	Phone-list
Kailey S.	Jeremy M.	Joey, Mario, Patrick	Aerin B.

- In one convenient location: All **9** [SeLo quizzes (pdf)](#).

- The cute **Home-A** was followed by the highly-praised **Class-A**, much admired for its two-tone format, and the comic relief of the Three Stooges.

- Tilable **Home-B (pdf)** was loads of fun.
  Computer-generated pictures of **Lmino Tilings (txt)**, illustrating a geometric induction-argument.
  The **domino-tiling sequence on 2xN** leads to the **roots of the problem**.
  The nifty **Class-B** graced its way into existence on Wedn., 24Oct2018.

- We went *One-on-One* with the cheery [Class-C](#), and we Won!
  - The dodecahedral IOP-D needed several pictures.
- Voila! our [SeLo syllabus](#).

Part of the **Spring 2017 SeLo** page:

[End-of-Semester Games Party](#) (click for larger)
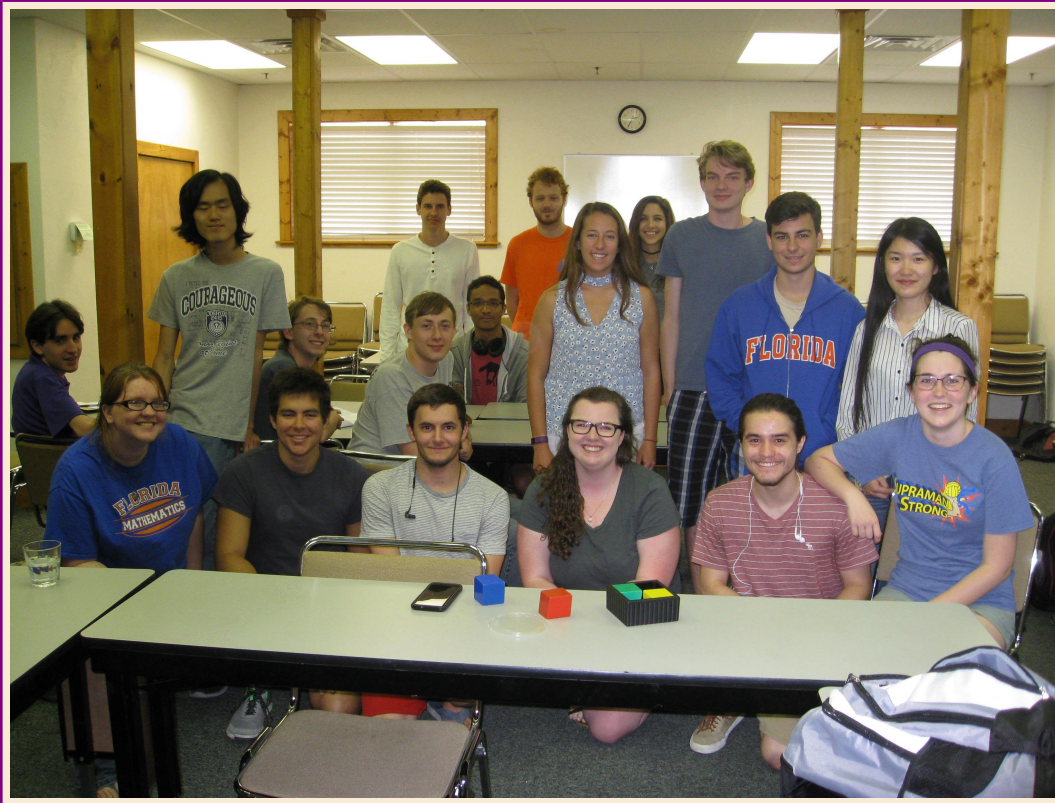
Math czars:

TIME	COMPUTER	BLACKBOARD	SPUR-PROBS	PHONE-LIST
Leah	Leah	Christopher P.	Nicholas C.	Lizzie(Donna)

- Our end-of-semester extra topic was: Introduction to the [Chromatic polynomial of a graph (pdf)](#).

- Available are all the [SeLo quizzes so far (pdf)](#).

- The thought-provoking [Home-V](#) also provoked great joy, with its three interesting essay questions.
  This was followed by the cute [Class-V](#).

- The multi-dimensional [Home-W](#), nicknamed *Waldo* inspired greatness.
  Folks thrilled to [Class-W](#) and its comforting question about Lminos, with which the cognoscenti were already familiar.

- [29Mar2017]: Applause showered [Class-X](#) [the last in-class exam of the semester, people tearfully observed], with its Nifty use of *Schroeder-Bernstein*, to prove a surprising result about the set of continuous functions.

- The [SeLo syllabus, with dates](#) is available.

- The intriguing IOP-Y was due 2PM, Thursday, 20Apr2017

Voila! some of the **Spring 2015, SeLo**, page:

Our [syllabus](#) gave an overview.
Our exams were:

- Team [W-home](#) and individual [W-class](#), the in-class component.

- Team [X-home](#) and individual [X-class](#).

- In-class [Y-class](#); there was *no* team component.

- Finally, there was an Individual *Optional* Project IOP-Z at semester's end.

Useful info from the **Spring 2014, SeLo**, page:

We had a [test of prerequisites](#) as well as [four microquizzes](#) during the semester.
The exams were:

- Team [W-home](#) and individual [W-class](#).

- Team [X-home](#) and individual [X-class](#).

- At semester's end, there was the *Optional* IOP-Y which had you do a Schroeder-Bernstein computation.

A section of the the **Autumn 2013 SeLo** page:

- Available are all the [SeLo quizzes (pdf)](#).

- Cheering crowds greeted the Gregorianish [Class-U (pdf)](#) with its 2-term linear recurrence and correct spelling of February ! Folks rushed to post solutions for their colleagues to admire.

"   "

- The delightful Home-V (pdf) was available *early*, for your team to solve. Home-V was due at the BoC [Beginning Of Class] on **Monday, 21Oct**. The cheerful individual component, Class-V (pdf) [Wedn., 23Oct.] was well-received by the Mathematical Intelligentsia, appreciating its tight connection to Home-V.

- Insightful, engaging, absorbing Home-W (pdf) became available the evening of **Tuesday, 12Nov.**. It was due at the BoC on **Monday, 18Nov.**. Class-W (pdf) the individual component, revealed itself on **Wednesday, 20Nov.**.

- Cardinaltiy arguments were cental to IOP-X [noon, Monday, 09Dec2013.]

- SeLo syllabus (txt).

Helpful Math czars.

TIME	COMPUTER	CHALK	ES-PROBS	PHONE-LIST
Rico	Corey	Sam-C	Alex	David

Part of the **Spring 2012 SeLo** page:

The various Math czars who help out:

TIME	COMPUTER	CHALK	BLACKBOARD	ES-PROBS	PHONE-LIST
Annie	Sofia	Stewart	Ian	Christopher M.	Alejandra

- [SeLo quizzes 2012g (pdf)](#).

- The magical [Home-W (pdf)](#) challenged your team's gray-cells with modular arithmetic. The take-home was due at BoC, on Monday, 06Feb2012. It nicely foreshadowed the [Class-W (pdf)](#), eliciting shouts of enthusiasm.

- Inspired by [Class-X (pdf)](#), the crowd cried "Give us more Zinc, Tin, Silver and Gold! —5 atoms; that's just not enough!" All this followed the nifty [Home-X (pdf)](#), which was due by noon, on Tuesday, 28Feb2012, slid completely under my office door.

- Folks found that [Class-Y (pdf)](#), was well-*Suited* to their bijective knowledge.

- Our IOP-Z had had an affine-code that students needed to decode!

- Voila: A [printable SeLo syllabus](#). Here is our [schedule](#).

Part of the **Spring 2011 SeLo** page:

Classroom czars who helped out:

ES-Probs	Computer	Time	Chalk	Blackboard	Phone-list

Mallory   Corey   David S.   Esther   Kristen   Blanko

(∃ another photo at bottom of page)

- Available are all the [quizzes so far (pdf)](#).

- Prior to the prereq [Class-T](#), you looked at the [sample test (pdf)](#).

- The highly-anticipated and outrageously upbeat [Class-U](#), entranced the cognoscenti with its cute use of the Gregorian calendar, and its nifty induction proof. While a few carped that it was "Too easy!", the general population raved about the exam's clean lines and minimalistic two-tone format. Critics were unanimous in their praise of questions that were slight alterations of earlier quiz and class conundra. A Smash Hit!, concluded all the major newspapers... and crys of "Encore!" and "Author! Author!" echoed throughout the Realm.

  - *N* travelers peregrenated through [Class-V](#) (Wed., 2Nov.), discovering that, starting at any question, they could visit *all* the questions. "*Better* than getting bitten by a rabid squirrel!", was the general consensus. (A minority opined "*Comparable* to rabid-squirrel bites —but with less fur...")

  - We liked [Class-W](#) so much, that we decided to spend an extra half-hour with it. Now *that* is one popular exam! (Taken, on Friday, 18Nov).

Here is part of the Autumn 2009 SeLo page:

The various Math czars who helped out:

COMPUTER	TIME	PHONE-LIST	CHALK
Marc	*Sigrun*	*Cara*	*John*

- Individual-Project Home-E was due

  1PM, Friday, 11Dec2009,

  carefully typed, but diagrams may be hand-drawn.

- Quizzes so far, and potential problems (pdf).

- Week-6: We continue in chapter 3. On Mon, 5Oct, we had extra classes, 9th&10th periods, in LIT127. We had our thought-provoking SeLo-B (pdf) on Wed, 7Oct., and here is a write-up of the congruence proof (pdf).

- Week-5: More on quantification. Introduced the **Powerset** operator. Please finish reading chapter 2 by Friday, 2 Oct.

- Week-15: Mon, Wed (7 and 9 Dec) are the last two days of class. Schroeder-Bernstein theorem.

- Week-14: Divisibility. Cardinality of sets, Cantor's diagonal argument. Project Home-E available Friday, 04Dec.

- Week-13: Monday: Further discussion of divisibility, and how the Euclidean algorithm applies to polynomials. (Wed, canceled; Fri is Thanksgiving vacation).

- Week-12:
  The fascinating [SeLo-D (pdf)](#) got rave reviews; the Crowd Clamored for More! [Wed, 18Nov.]
    Some examples of [computer generated Lmino tilings (txt)](#).

- Week-11: *Examples of induction:* Fibonacci numbers, and 2 base cases. Every posint factors as a product of primes irreducibles (existence). All horses have the some color. Lmino tiling.

- Week-10: Defns of a **lattice**, a particular kind of poset (partially-ordered set). Strong/weak induction. *Non-constructive proof:* There exists positive irrational numbers B,E such that B^E is rational.

- Week-9: Read 4.3.
  We had the stimulating [SeLo-C (pdf)](#) in class on Wed, 28Oct.
    Hopefully, Eager Mathematicians rush to post Solutions….

- Week-8: Vacuous operations. What means "the empty sum", "empty product" "empty max" "empty gcd"?

- Week-7: Please finish reading sections 3.1–3.7. For 26Oct, have read 4.1, 4.2.
  Indexed and non-indexed big-operators.
    Decimal notation and "repeated decimals".
    Binomial and multinomial coeffs. Proof of **Fermat's Little Thm** by induction. Using a binomial coeff to count the number of ways of choosing N objects out of T distinguished types.
    There was a [makeup SeLo-B (pdf)](#) for those with a legitimate reason for missing the original; please post solns.

- Week-4: Please cogitate deeply over [iterated LBolt & linear congruences (pdf)](#).
  We: Started *Quantifiers* and reviewed *Free variables* and these functions: **d(), sigma(), EulerPhi(), floor(), ceiling()**. Discussed notations for tuples/sequences, gcd of tuple or set, relation between *contrapositive*, *converse* and *inverse* of a stmt.

- Week-3:
  We start *Propositional logic* (also called *sentential logic*). Play with the [Venn-diagram self test](#), noting that this page uses B' to mean the complement of B, which we generally write a B^c. [ASIDE: Please read our [general terminology (pdf)](#).]
    We'll also look at the [w:Euclidean Algorithm](#) (i.e, the "Lightning Bolt algorithm"). The [LBolt frame (pdf)](#) has seven practice problems on page 1 [[LBolt answers (txt)](#) are available], and six "make your own problem" on page 2.
    Please grok completely how to [easily solve a linear congruence (pdf)](#).

- Week-2: Please print and read [Mersenne primes and Even Perfect numbers (pdf)](#).
  We proved that **Primes** has arbitrarily long gaps. We proved *Euclid's thm* that there are infinitely many primes.
    Having defined the arithmetic progression AP(s,G) := [s + GZ], we stated *Dirichlet's thm* for coprime APs. We noticed that Euclid's thm is the special case AP(0,1) of Dirichlet's. We proved Dirichlet's for AP(-1,4), and Prof. King gave an exercise to prove the same for AP(-1,3), and AP(-1,6).
    We defined [w:Modular arithmetic](#) and proved that addition/subtraction and multiplication are preserved, mod N.

- Week-1: Primes and Mersenne primes and [w:Perfect numbers](#).

- Week-0:
  David Gale's Game of "[chomp](#)" in [w:Wikipedia](#). Doron [Zeilberger's](#) "[Three-rowed Chomp](#)".
    John posted some solns to our prerequisite [mini-exam SeLo-A (pdf)](#).

Please take a gander at our [Syllabus (txt)](#) and [Past courses with notes, exams and links](#).

Material from the Spring 2008 SeLo page:

Our textbook was *Sets, Functions and Logic (**Third** Edition)*.

AUTHOR: Keith Devlin   ISBN: 978-1584884491

YEAR: 2003   PUBLISHER: Chapman & Hall/CRC

*T*HE MATH CZARS WHO HELPED OUT.

E-PROBS	COMPUTER	PHONE-LIST	CHALK	BLACKBOARD	TIME
Karly & Josh	Michael	Kyle	Vincent	Ben	Ross

- Applause and acclaim was the response to the gorgeous Prereq-A (pdf) with its clean lines, and minimalistic two-tone format; this was the test of prerequisite knowledge.

- General gaity greeted the taut Class-B (pdf) with its reference to Lewis Carroll and its nice mix of problems.

- Here is the practice exam for Class-C (pdf), together with the actual Class-C (pdf). Rumor has it that this exam was the right length :-)

- Many folk finished Class-D (pdf) early, so as to go home and read about Completeness in Chapter 5. I invite you to post solutions to our archive.

  Voila a practice exam for Class-D (pdf). It is quite similar to, but longer, the actual Class-D.

- Individual final-project E (pdf), was due,
  2PM, Friday, 25Apr2008,
  carefully typed.