


2 Sets and Logic

A bout of Nostalgia? See [past SeLo courses](#).

Welcome

Our [Teaching Page](#) has important information for my students. (It has the [Notes, Exams and Links](#) from all of my previous courses.)

The *Teaching Page* has **my schedule**, **LOR** guidelines, and [Usually Useful Pamphlets](#). One of them is the [Checklist](#)  (pdf) which gives pointers on competent mathematical writing. Further information is at our **class-archive URL** (I email this private URL directly to students).

In all of my courses, **attendance is absolutely required** (excepting illness and religious holidays). In the unfortunate event that you miss a class, *you are responsible* to get all **Notes / Announcements / TheWholeNineYards** from a classmate, or several. All my classes have a **substantial class-participation grade**.

Assignment for first week of SeLo: (See below, for the materials referred to.)

- To help you self-evaluate, take 90 minutes to solve as many problem as you can, on this [test of high-school mathematics, with a touch of calculus \(pdf\)](#).
- Learn the [Math-Greek alphabet \(pdf\)](#)
- *Work through* BoP, sections 1.1 through 1.9. Write-up (but do not hand-in) at least 3 HW problems from pages 1-14.
- In *PList*., read pages 1-6, memorize abbreviations in *Appendix: Notation*.
- Exams from previous [SeLo incarnations](#):

This will help you decide if my teaching-style is the right style for you.

- Read [Set-builder notation \(W\)](#) (up through “Equivalent predicates...”), becoming comfortable with the notation.
- In *SaP* (“Structure and Proof”) work through pages 11-17.

Important: For us, the (double-bar N) symbol $\mathbb{N} = \{0, 1, 2, \dots\}$; i.e *zero is a natural number, a natnum*. This is also the convention in *SaP* but, unfortunately, not the convention in *BoP*.

So when you read \mathbb{N} in *BoP*, replace it with $\mathbb{Z}_+ = \{1, 2, 3, 4, \dots\}$; the set of *positive integers*; the *posints*.

Our two, free, online texts (you can freely download the PDFs to your computer) are these:

Main textbook: The [Book of Proof \(BoP\)](#), by [Richard Hammack](#).

Secondarily, we will use

[Transition to Higher Mathematics: Structure and Proof \(SaP\)](#), by [Bob A. Dumas](#) and [John E. McCarthy](#).

SeLo Resources

- *Memorize!* the [Math-Greek alphabet \(pdf\)](#).
- **The PList:** (Problem List for SeLo has [hyperlinks](#) in the [Table-of-Contents](#) and the [Index](#).) ~~UPDATED~~ [Date].
- [Does Zero = One? \(pdf\)](#). Here are some ~~proofs~~ *poofs* about which you can post to our Archive.
- *Look Ma!* All 0 SeLo quizzes so far (pdf) ~~UPDATED~~ [Date]
- Optional: [Practice: Binomials, complex arithmetic](#).
- **Near future:** Please work-through [W: Euclidean algorithm](#) (up through “Extended Euclidean...” but skip the proofs) *and* work-through [W: Modular arithmetic](#) (through “Applications”).
The **Euclidean algorithm** can be conveniently applied in table-form; I call this form “Lightning Bolt ” because the update-rule looks like a lightning-bolt (used thrice). *Please read* the [Lightning-bolt algorithm \(pdf\)](#), learning the algorithm, but skipping the proofs.
Suggestion: Print out on paper (yes, *actual paper*), the [practice sheet for LBolt \(pdf\)](#) and fill-in the tables.
- **Near future:** The first page of  [Algorithms in Number Theory \(pdf\)](#), uses LBolt iteratively to compute the GCD of a list of integers, together with its list of Bézout multipliers. Page 2 uses LBolt to solve linear congruences: “Find all x where $33x \pmod{114}$ congruent to 18.”
- **Optional:** Examples of [fusing congruences \(txt\)](#) using LBolt.
Everybody loves the *Euler-Fermat thm*. Available is [Using EFT to solve \$102^{70} + 1 = 113 = b^{37}\$ \(txt\)](#), from Prof. William Stein's book.
- A std proof of the [Inclusion-Exclusion principle \(pdf\)](#), together with *Candy-Store*, *Derangement* and *Stirling-number* examples.
- An [Introduction to Isomorphism, via Gambling](#). It asks: *Why 2, when 76 seems correct?*
- What is [Hall's Marriage \(Matching\) Lemma](#)?
- **Future:** Our [Primer on cardinality](#).
In addition to proof by *raster scan*, we can prove that $N \times N$ is equinumerous with N via [Boustrophedon](#), which can even be [pronounced!](#)
- **Optional:** Our [Primer on Polynomials](#) has further information on Algebraic Numbers, for the *Curious Ambitious Student*.
- **Future, optional:** [Number Theory grab-bag \(pdf\)](#). (I wrote this for a NT class, so we'll need to skip parts.) **Optional:** A proof of the [Chinese Remainder Theorem \(pdf\)](#) [CRT], as a ring-isomorphism thm. Proves that Euler phi is a multiplicative fnc. An example of [using CRT to count roots of a polynomial](#)
- [2022g SeLo syllabus](#).

Web resources

- [Ring Basics \(pdf\)](#).
- Fun, challenging problems: [IMO](#) and [USAMO](#) and [HMMT](#) and [Putnam](#).
- The famous [On-line Encyclopedia of Integer Sequences](#), and some [W: OEIS history](#), and a [video with a challenge at the end](#).
- Truth-table displayer:
- A free site, [Merge PDF](#), for merging multiple PDFs into one. I've used this and it worked fine. There are other free ones on the web as well. Use whatever works for you.
In case you want it, this free [as of 21Dec2020]. site [converts PDFs to PNGs](#).
- **Tips on writing proofs**
 - ["How To Write Proofs" \(html\)](#), by Prof. Larry W. Cusick, [INTERMEDIATE]. Examples mostly from Elem. Number Theory; some from Calculus.
 - [Prof. Christopher Heil's page \(pdf\)](#) [4 pages, INTRO]. A well written survey of the structure of proofs. Has one example of induction (recursion).
- **Optional:** A cute proof that [e is irrational](#).

Quantifiers \forall and \exists ("for all" and "there exists") are like nitroglycerin, in that one little mis-step leads to the whole thing blowing up in your face.

There is no partial credit when it comes to Explosives and Quantifiers.

-JLF King

Ending in style

- Prof. King's [Mastery of Zoom](#) [except for the cigarette]. [Source unknown]
- An [End-of-Semester Math Derivation](#).

[JK Home page](#)

