MWF 8th period (3:00-3:50pm) – LIT 207 – FALL 2017 – Section 18D2

Krishnaswami Alladi

304 Little Hall

(352) 294-2290
email: alladik@ufl.edu

M and W 6th period (12:50 – 1:40 pm) in LIT 304 and by appointment.

The study of irrational numbers dates back to Greek antiquity. Yet the subject remains an active area of research today. Although "almost all" real numbers are irrational, it is very difficult to establish the irrationality of a given number. We will study a variety of techniques which will help in confirming the irrationality of different classes of numbers starting with Dirichlet's fundamental criterion for irrationality. Following this, we will discuss irrationality criteria utilizing series and product representations of reals due to Engel, Cantor, and Sylvester, regular and general continued fraction expansions, and Farey fractions. We will present various proofs of the irrationality of important numbers such as e and π, and the irrationality of functions like the trigonometric, hyperbolic, and the Bessel functions at rational arguments. Next we will take up the study of the closeness of approximation of irrationals by rationals, and introduce the concept of irrationality measures. This is the subject of Diophantine approximations. It will be shown that the truncations of the regular continued fraction expansion of real numbers generate the sequence of "best approximations", but it is very difficult to determine the continued fraction expansion of a given irrational. We will develop methods to obtain effective irrationality measures. This will involve the use of Legendre polynomials and Pade approximations. An important outcome of this approach is the irrationality of the Riemann zeta function at the odd integer 3 – a fact conjectured long ago but established only in 1978. The study of irrationality measures also explains why Pell's equation such as $x^2 – 2y^2 = 1$ has infinitely many solutions whereas the Thue equation $x^3 – 2y^3 = 1$ has only a finitely many integer solutions. Thus the subject of Diophantine approximations is closely associated with the theory of Diophantine equations. As part of our study of Diophantine approximations, we will include a discussion of uniform distribution and of normal numbers – numbers whose digits are statistically uniformly distributed. Finally we will launch the study of transcendental numbers by showing first that e is transcendental, and more generally that the result of Lindemann that the exponential function takes transcendental values at nonzero algebraic arguments. Since exp(iπ) = -1, Lindemann's theorem implies that π is transcendental, thereby showing the impossibility of squaring the circle – one of the three problems of Greek antiquity. We also hope to discuss measures of transcendence and the transcendence of some fascinating functions (defined by lacunary series) at algebraic arguments. A host of great problems remain unsolved: Is Euler's constant γ irrational? Are the values of the Riemann zeta function at odd integers > 3 irrational? What is the precise irrationality measure for π? We know that "almost all" numbers are normal to all bases, but we do not know a single example of a number whose digit expansion to all integral bases (not just base 10) are normal. The course will be completely self-contained and can be followed by any of our graduate students as well as those in allied disciplines such as computer science and statistics. I will use my own detailed notes but will give a number of books as references. The list of topics given above is vast, and I will cover as many of them as time permits. My goal is not to race through topics, but to do a thorough discussion at a pace that is comfortable. If there is interest, the course will be extended into Spring 2016. The course grade will be based on homework and/or a seminar that the student could give.

No assigned text. I will use my own notes. A number of texts will be given as references.

Grades will be based on a few homework assignments, and seminars that students will have an opportunity to give.

Students requesting classroom accommodation must first register with the Dean of Students Office. The Dean of Students Office will provide documentation to the student who must then provide this documentation to the Instructor when requesting accommodation.

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