



The topics are divided into six general areas:

1. Group Theory
 - Nilpotent groups
 - Free groups
 - Linear groups
2. Category theory
 - Categories, subcategories
 - Functors, equivalence of categories
 - Adjoint functors
 - Universal properties, representability
3. Galois theory
 - Algebraic closure
 - Algebraic, normal and separable extensions
 - Galois correspondence (finite extensions)
 - Solvability of equations
 - Cubic and quartic equations; cyclotomic fields
4. Field theory
 - Algebraic and transcendental extensions
 - Transcendence basis of an extension
5. Commutative ring theory
 - Localization; support of a module
 - Spectrum of a commutative ring
 - Noetherian and Artinian rings
 - Hilbert Nullstellensatz
 - Hilbert Basis Theorem
 - Integral extensions; integral closure
 - Associated primes of a module
 - Discrete valuation rings; Dedekind domains
 - Projective, injective and flat modules; invertible ideals
6. Noncommutative ring theory
 - Tensor products
 - Tensor, symmetric and exterior algebras
 - Primitive rings; density theorem
 - Semisimple rings
 - Wedderburn's theorem on finite division rings

Bibliography:

In recent years, one of the following has served as the core text for the course.

1. David S. Dummit and Richard M. Foote, *Abstract Algebra*, 3rd edition, Wiley
2. Thomas W. Hungerford, *Algebra*, Springer Graduate Texts in Mathematics 73
3. Serge Lang, *Algebra*, Springer Graduate Texts in Mathematics 211

Supplementary material can be drawn from the following books.

1. Irving Kaplansky, *Fields and Rings*,
(supplementary material for Galois theory, particularly cubic and quartic equations and cyclotomic fields; supplementary material for noncommutative rings, particularly Noetherian and Artinian rings, the Hilbert Nullstellensatz and Hilbert Basis Theorem)
2. Hideyuki Matsumura, *Commutative Ring Theory*, Cambridge University Press (supplementary material in the area of commutative ring theory, particularly the first two and last three subtopics)

Additional Resources:

- [Richard Foote's page of errata for Dummit and Foote](#)
- [George Bergman's Companion to Lang's Algebra with errata](#)
- [GMA page on past PhD Algebra Exams](#)

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