Jindrich Zapletal
Department of Mathematics

Descriptive Set Theory Spring 2019

Course.

This is the web page for course MAT6932 section 3F92 five digit 17210, meeting MWF 8th period in LIT233.

Instructor.

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Course contents.

The course covers the part of descriptive set theory which deals with the classification of Borel and analytic equivalence relation according to their complexity. It turns out that most equivalence problems encountered in analysis, algebra, dynamical systems etc. fit into this context. Therefore, our technology will allow us to compar (say) the problem of homeomorphism of compact metrizable spaces with the problem of isomorphism of countable groups in a sensible and informative way. The theory has been initiated in the early 1990's, recorded many notable successes, and it is a very active field of research today.

During Week 1, I will introduce Borel reducibility of analytic equivalence relations, the basic known features of this quasiorder, basic examples, and the universal analytic equivalence relation. The remainder of the course is divided into four blocks.

Block 1.

Smooth equivalence relations. [These are the equivalence relations Borel reducible to the identity; in other words, the equivalence relations which allow an assignment of a complete numerical invariant to equivalence classes.] Week 2. Basic examples. Week 3. The Silver dichotomy and Vaught's conjecture. Week 4. Glimm–Effros dichotomy. [There is a simple equivalence relation which is not smooth, and it is the canonical obstacle.]

Block 2.

Orbit equivalence relations. Week 5. Topological groups, the Birkhoff–Kakutani metrizability theorem.

[Second countable topological groups have an invariant metric inducing their topology.] Week 6. Examples of Polish groups, universal Polish groups. [Permutation groups, groups of homeomorphisms and isometries and such.] Week 7. Actions of Polish groups and their orbit equivalence relations. Examples, universal actions. Week 8. E1 conjecture. [There is a simple equivalence relation which is not reducible to an orbit e.r., but is it the canonical obstacle?]

Block 3.

Equivalence relations with countable classes. [Typically called countable Borel equivalence relations (CBER) by an abuse of terminology; very common class.] Week 9. Feldman–Moore theorem. [All of them are orbit equivalence relations of actions of countable groups.] Week 10. Examples. Hyperfinite equivalence relations, universal CBER. [The simplest CBERs can be written as an increasing union of equivalence relations with finite classes, and one can classify all of these.] Week 11–12. Amenable groups and their orbit equivalence relations, Banach–Tarski paradox, Martin conjecture. [An effort to find a CBER which is not hyperfinite leads to considerations related to those at the root of the Banach–Tarski paradox: it is possible to divide the unit ball into finitely many pieces and rearrange these with rigid motions into two copies of the unit ball (?!?)]

Block 4.

Equivalence relations classifiable by countable structures. [A most common type of classification of equivalence classes attempted by mathematicians is by countable structures, such as groups, up to ismomorphism.] Week 13. Model theory. [Learning how to define sensible classes of countable structures and discern between them.] Weeks 14–15. Turbulence. [A key method for proving that an orbit equivalence relation is not classifiable by countable structures. Many applications.]

Grading.

After each of the four blocks, I will assign a take home exam. The three take home exams will be equally weighted.

Textbook.

The textbook is Su Gao: Invariant Set Theory Another possibility is Kanovei: Borel equivalence relations

Further administrative matters.

Requirements for class attendance and make-up exams, assignments, and other work in this course are consistent with university policies that can be found in the online catalog at: https://catalog.ufl.edu/ugrad/current/regulations/info/attendance.aspx.
Students are expected to provide feedback on the quality of instruction in this course based on 10 criteria. These evaluations are conducted online at https://evaluations.ufl.edu. Evaluations are typically open during the last two or three weeks of the semester, but students will be given specific times when they are open. Summary results of these assessments are available to students at https://evaluations.ufl.edu/results/.
Students requesting classroom accommodation must first register with the Dean of Students Office. The

Dean of Students Office will provide documentation to the student who must then provide this documentation to the Instructor when requesting accommodation.

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